Maximum Efficiency and Output of Class-F Power Amplifiers

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***Abstract*—A class-F power amplifier (PA) improves efficiency and power-output capability (over that of class A) by using selected harmonics to shape its drain–voltage and drain–current waveforms. Typically, one waveform (e.g., voltage) approximates a square wave, while the other (e.g., current) approximates a half sine wave. The output power and efficiency of an ideal class-F PA can be related to the Fourier coefficients of the waveforms, and Fourier coefficients for maximally flat waveforms have been determined. This paper extends that theory by determining the coefficients for the maximum power and efficiency possible in a class-F PA with a given set of controlled harmonics.**

***Index Terms*—Amplifier, class F, power.**

I. INTRODUCTION

THE implementation of class-F power amplifiers (PAs) is based upon achieving (approximately) open- or short-circuit impedances at the harmonic frequencies [1], [2], [12]. Consequently, it has become a popular technique for improving the efficiency of PAs operating at UHF and microwave frequencies. While a wide variety of different class-F PAs has been implemented [3]–[9], the impact of using different numbers of harmonics remains only partially understood. Previous papers by the author determine performance parameters for maximally flat waveforms [10], [11]. The purpose of this paper is to expand upon that work to determine the upper limits of output power and efficiency as functions of the number of harmonics used in the amplifier. This allows a designer to make a tradeoff between output-network complexity and efficiency.

II. BASIC THEORY

The circuit of a generic class-F PA is shown in Fig. 1. The basic principles of operation are as follows.

- Fundamental-frequency drain voltage and current are shifted in phase by 180° from each other.
- One drain waveform (e.g., voltage) adds odd harmonics to build its shape to a square wave (Fig. 2).
- The other drain waveform (e.g., current) adds even harmonics to build its shape toward a half sine wave (Fig. 3).
- No power is generated at the harmonics because there is either no voltage or no current present at a given harmonic. Harmonic impedances are either zero or infinite.

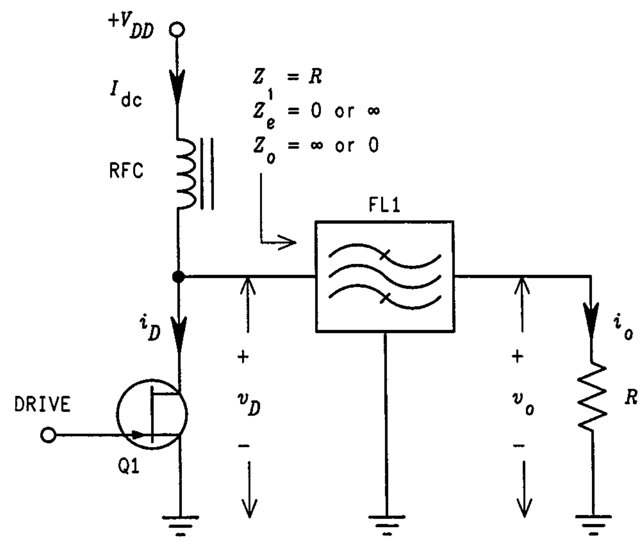

Fig. 1. Generic class-F PA.

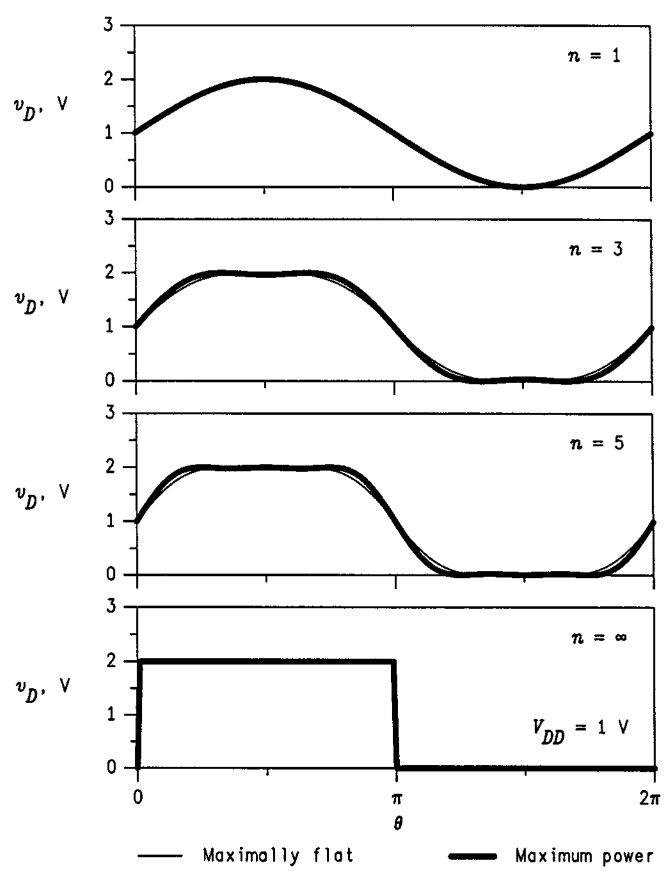

Fig. 2. Voltage (odd-harmonic) waveforms.

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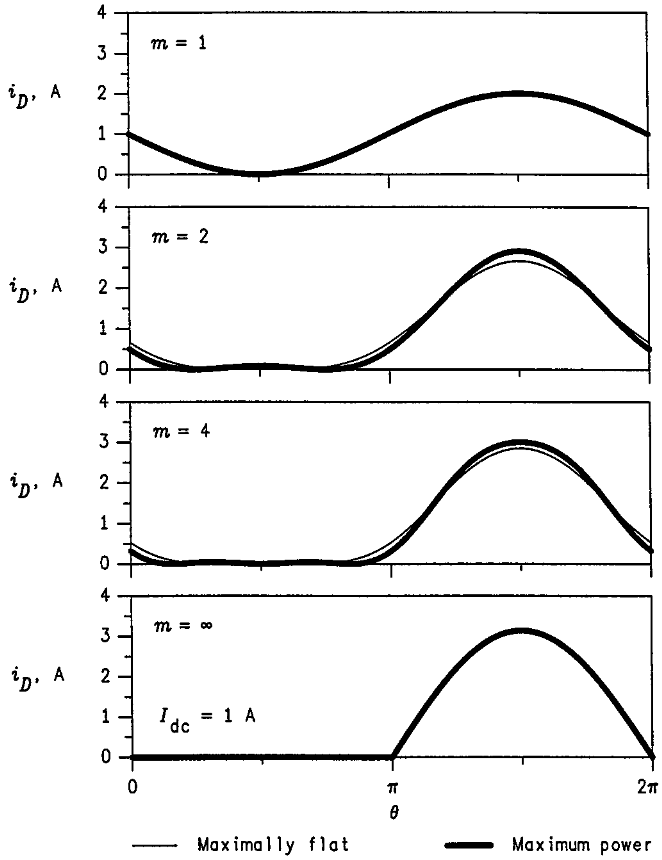

Fig. 3. Current (even-harmonic) waveforms.

- At microwave frequencies, the number of harmonics is usually relatively small.
- The RF-power device acts as a saturating current source (e.g., a soft switch). Only when all harmonics are properly terminated can it act as a true switch. Negative voltage and current are not permitted.

Since waveforms contain either odd or even harmonics and a given harmonic is present only in one waveform, the effects of the odd and even harmonics can be determined separately. The effects of the harmonics in a given waveform are manifested in waveform factors q_V and γ_I [10] that relate the dc components (V_{DD} and $I_{\rm dc}$) to the fundamental-frequency components (V_{om} and I_{om}), and peak drain voltage and current ($v_{D\,\max}$ and $i_{D\,\max}$) as follows:

$$V_{om} = \gamma_V V_{DD} \tag{1}$$

$$v_{D\,\max} = \delta_V V_{DD} \tag{2}$$

$$I_{om} = \gamma_I I_{\rm dc} \tag{3}$$

$$i_{D\,\max} = \delta_I I_{\rm dc}. \tag{4}$$

A resistive load impedance at the fundamental frequency is assumed, as is an ideal FET (no on-state resistance, no distortion of the waveform).

The power output, dc input, and efficiency are then

$$P_o = \frac{V_{om}^2}{2R} = \frac{\gamma_V^2 V_{DD}^2}{2R} \tag{5}$$

$$I_{\rm dc} = \frac{I_{om}}{\gamma_I} = \frac{V_{om}}{\gamma_I R} = \frac{\gamma_V V_{DD}}{\gamma_I R} \tag{6}$$

and

$$\eta = \frac{P_o}{P_i} = \frac{\gamma_V \gamma_I}{2}. \tag{7}$$

The power-output capability (output power with $v_{D\,\max} = 1$ V and $i_{D\,\max} = 1$ A) is obtained by dividing power output by peak voltage and current, which yields

$$P_{\max} = \frac{P_o}{v_{D\,\max} i_{D\,\max}} = \frac{\gamma_V \gamma_I}{2\,\delta_V\,\delta_I} = \frac{\eta}{\delta_V\,\delta_I}. \tag{8}$$

As shown in [10], the peak-to-peak of the even (current) waveform is unaffected by properly phased harmonics, hence, $P_{\max} = \gamma_V/8$.

III. Coefficients for Maximum Power and Efficiency

Maximally flat waveforms represent the limiting case of perfect (low resistance) saturation of the RF-power device during conduction of peak drain current. The waveform factors for maximally flat waveforms are derived in [10] and [11] by setting various derivatives of the waveforms to zero. The resulting power and efficiency, while typical, are not the maximum possible.

To find the Fourier coefficients for maximum power and efficiency, it is convenient to fix the fundamental-frequency amplitude at unity. The amplitude of the harmonic(s) is then adjusted to minimize the downward excursion of the waveform. Fixing the waveform minimum at zero gives the minimum supply voltage needed for full output, which, in turn, minimizes the dc-input power and, therefore, maximizes efficiency. Flattening of the waveform reduces the peak voltage and, therefore, maximizes the power-output capability for a given rating. Thus, maximum efficiency and maximum power-output capability occur for the same waveform coefficients.

The maximum efficiency and power-output capability for a given set of harmonics (Table III) is obtained by inserting the appropriate waveform coefficients from Tables I and II into (7) and (8), respectively. As shown in Fig. 4, the maximum efficiency of an ideal PA increases from the 50% of class A to 70.7, 81.6, 86.6, and 90.4 as harmonics are added.This process is readily illustrated with a waveform that is enhanced by only the second harmonic

$$v(\theta) = V_{DD} + \sin\theta - b\cos 2\theta. \tag{9}$$

The derivative with respect to time is

$$dv(\theta)/d\theta = (\cos\theta)(1 + 4b\sin\theta). \tag{10}$$

To find the location of the minimum voltage, (10) is set equal to zero, which produces

$$\sin\theta_m = -1/4b \tag{11}$$

and

$$\cos^2\theta_m = 1 - 1/16b^2. \tag{12}$$

TABLE I
MAXIMUM-EFFICIENCY WAVEFORM COEFFICIENTS FOR ODD HARMONICS

HARM	δ_V	$\gamma_V = V_{om}/V_{DD}$	V_{3m}/V_{om}	V_{5m}/V_{om}
1	2	1	-----	-----
3	2	$2/3^{1/2} = 1.1547$	1/6 = 0.1667	-----
5	2	1.05146	-----	-0.06180
3+5	2	1.2071	0.2323	0.0607
∞	2	$4/\pi = 1.273$	$4/3\pi = 0.424$	$4/5\pi = 0.255$

TABLE II
MAXIMUM-EFFICIENCY WAVEFORM COEFFICIENTS FOR EVEN HARMONICS

HARM	δ_I	$\gamma_I = I_{om}/I_{dc}$	I_{2m}/I_{om}	I_{4m}/I_{om}
1	2	1	-----	-----
2	$3/2+2^{1/2}=2.9142$	$2^{1/2}=1.4142$	$2^{1/2}/4=0.3540$	-----
4	2.1863	1.0824	-----	-0.0957
2+4	3.0000	1.5000	0.3890	0.0556
∞	$\pi = 3.142$	$\pi/2 = 1.571$	2/3 = 0.667	2/15 = 0.133

TABLE III
MAXIMUM EFFICIENCY AND POWER-OUTPUT CAPABILITY OF CLASS-F PAs

m	η			
	$n = 1$	$n = 3$	$n = 5$	$n = \infty$
1	1/2 = 0.500	$1/3^{1/2} = 0.5774$	0.6033	$2/\pi = 0.637$
2	0.7071	0.8165	0.8532	0.9003
4	0.7497	0.8656	0.9045	0.9545
∞	$\pi/4 = 0.785$	0.9069	0.9477	1 = 1.000
	P_{max}			
	$n = 1$	$n = 3$	$n = 5$	$n = \infty$
	1/8 = 0.125	$1/4 \cdot 3^{1/2} = 0.1443$	0.1508	$1/2\pi = 0.159$

The supply voltage in (9) must be sufficient to ensure that the drain voltage is nonnegative, thus

$$V_{DD} = -\sin\theta_m + b\cos 2\theta_m = b + \frac{1}{8b}. \tag{13}$$

To minimize V_{DD}

$$\frac{dV_{DD}}{db} = 1 - \frac{1}{8b^2}. \tag{14}$$

Setting this equal to zero produces

$$b = \frac{1}{8^{1/2}} = \frac{2^{1/2}}{4} = 0.3535. \tag{15}$$

The waveform constants are then determined from

$$V_{DD} = b + \frac{1}{8b} = \frac{2^{1/2}}{2} = 0.707. \tag{16}$$

For $V_{om} = 1$, $V_{2m} = 2^{1/2}/4 = 0.35 = V_{DD}/2$. The waveform coefficient for the fundamental-frequency amplitude is

$$\gamma_V = \frac{V_{om}}{V_{DD}} = \frac{1}{2^{1/2}/2} = 2^{1/2} = 1.414 \tag{17}$$

and the second harmonic amplitude is

$$\frac{V_{2m}}{V_{DD}} = \frac{2^{1/2}/4}{2^{1/2}/2} = \frac{1}{2}. \tag{18}$$

The peak voltage is

$$v_{D\,\max} = V_{DD} + V_{om} + V_{2m} \tag{19}$$

and, hence,

$$\delta_V = \frac{v_{D\,\max}}{V_{DD}} = \frac{2^{1/2}/2 + 1 + 2^{1/2}/4}{2^{1/2}/2} = \frac{3}{2} + 2^{1/2} = 2.914. \tag{20}$$

The coefficients for the third harmonic are derived in an analogous manner [3]. Those for the fifth harmonic by itself can similarly be derived with the aid of a computer algebra program (Maple). The derivation for the amplitude of fourth harmonic by itself degenerates to a numerical solution in the final step.

The coefficients for combinations of harmonics (second and fourth, third and fifth) can, in principle, be obtained in a generally similar manner. Derivatives of the minimum voltage must be taken with respect to both harmonic amplitudes and the resultant equations solved simultaneously. Unfortunately, an analytical solution was not found, even with the aid of a computer algebra program. These sets of coefficients are, therefore, obtained by a numerical evaluation based upon the waveforms themselves.

The resultant coefficients are given in Tables I and II, and the corresponding waveforms are shown in Figs. 2 and 3. In contrast to the maximally flat waveforms, the maximum power/efficiency waveforms exhibit slight ripples.

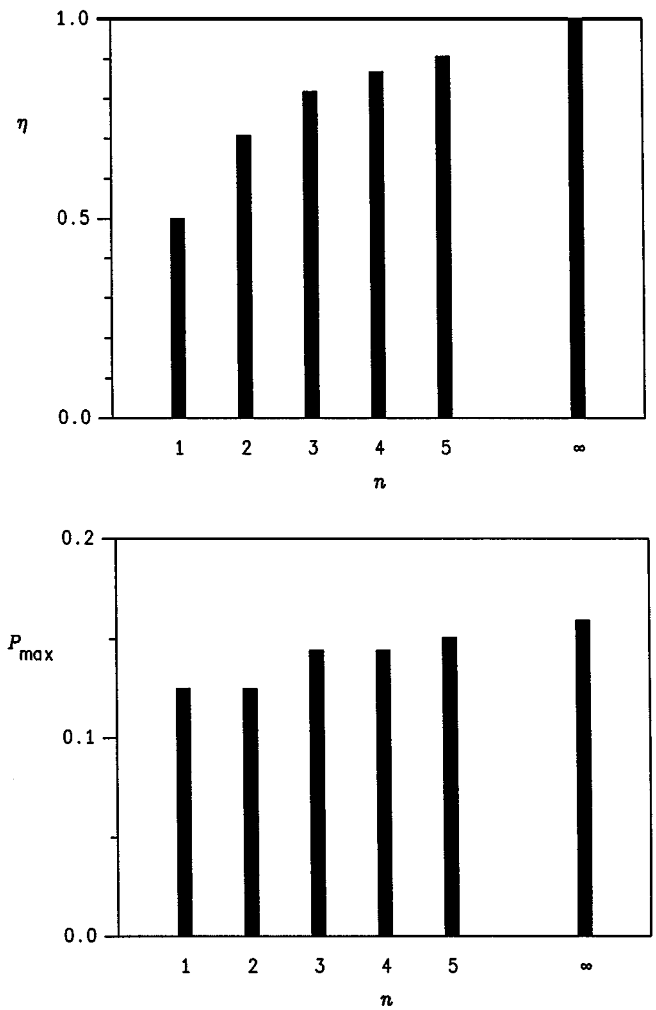

Fig. 4. Efficiency and power-output capability.

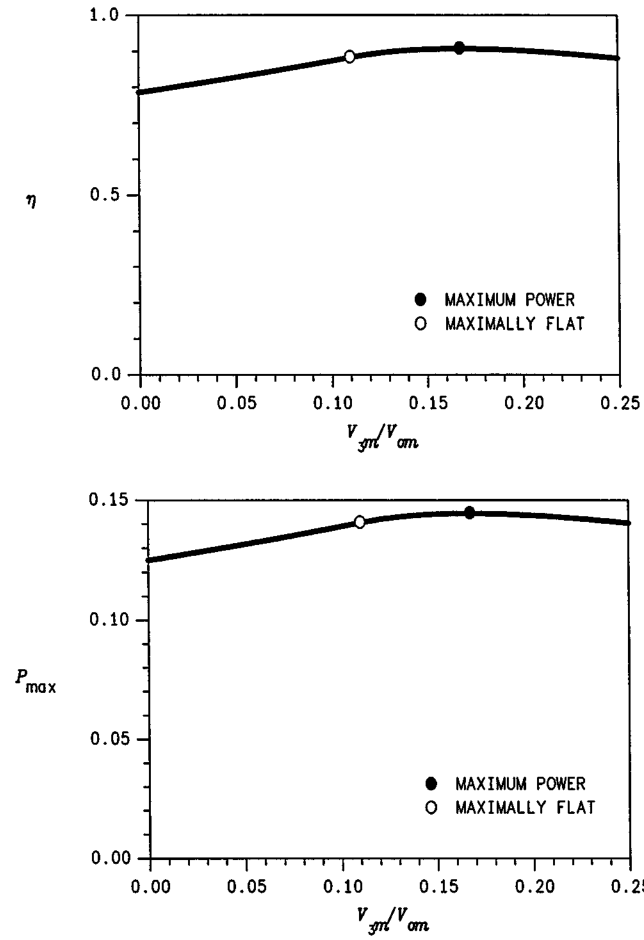

Fig. 5. Effect of third-harmonic level.

IV. Maximum Power and Efficiency

The variations of efficiency and power-output capability with the level of the third harmonic are shown in Fig. 5. The variation is gradual and relatively flat near the maximum-efficiency point, hence, harmonic levels are not especially critical. The maximum efficiency and power-output capability are generally 6%–8% higher than those for maximally flat waveforms.

V. Conclusions

Waveform coefficients for maximum efficiency and power-output capability in a class-F amplifier have been determined. The resultant efficiencies and power-output capabilities are the absolute maxima possible in an ideal class-F PA with a finite number of harmonics. These results are scalable to real PAs by inclusion of appropriate factors for on-state resistance and load reactance [1]. A subsequent paper will show that the same maximum efficiencies apply to class-C and class-E PAs based upon a finite number of harmonics.

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